

# COLUMN AND ARCH BUCKLING ACCORDING TO THE THEORETICAL , NUMERICAL, EXPERIMENTAL AND EUROCODE STUDIES FOR EDUCATIONAL PURPOSES

Aarne Jutila, Professor em., Dr Tech., Dr h.c .\*, Bertalan Szabó, Associate Professor, PhD, Dr Tech. \*\* & Miklós Iványi, Professor, Dr.Sc. \*\*\*

\* Extraplan Oy, Espoo, Finland  
E-mail: [aarne.jutila@extraplan.fi](mailto:aarne.jutila@extraplan.fi)

\*\* Budapest University of Technology and Economics, Department of Structural Engineering, Hungary

E-mail: [bszabo@epito.bme.hu](mailto:bszabo@epito.bme.hu)

\*\*\* Pécs University, Pécs, Department of Strength of Materials, Hungary  
E-mail: [steelivanyi@epito.bme.hu](mailto:steelivanyi@epito.bme.hu)

**Keywords:** bridge, column and arch buckling, education, Eurocode, Euler critical load, experimental results, numerical methods.

**Abstract:** *The paper describes how column and arch buckling problems are handled by using different methods and how the results should be interpreted to get a reliable understanding of the phenomenon and its physical behaviour. The issue mentioned last is of main importance, when buckling problems are introduced to young engineers with little experience. In reality, they normally prefer either numerical methods or rely on codes without knowing, what they really are calculating. In many cases also the codes are written in such a form that the real physical behaviour cannot be followed. An improvement would be, if the mathematical formulation of the code procedure and equations at least were presented in another way and the symbols familiar from theoretical studies were used. The paper tackles also such issues with a suggestion for future codes dealing with the problems presented in the paper.*

*In the present paper, theoretical studies are carried out by using traditional methods of stability. In the numerical calculations linear methods are used. The experimental study results are taken from tests carried out by the authors. The code studies are based on the relevant Eurocodes and the safety factors related to them are used.*

*The main conclusion of the paper is that a stability process varies a lot depending on which method is used: analytical, numerical, experimental or design codes. Their differences make it difficult to get an overall view on them. They also lead to different results, which increases the engineers responsibility to verify the applicability of the solution. In all cases the model selection is of main importance*

## 1. INTRODUCTION

In the modern design practice structures and structural components are getting slender and slender because of availability of more and more sophisticated design programs. At the same time structural codes are growing in volume, but not definitely in coverage of the problem field. This puts increased demands on the designer and especially on education of young engineers. Should one rely on codes covering only simple cases or should one use programs, whose basics and limitations are not known to the designer, especially to a young structural engineer?

For understanding the real structural behaviour in stability problems theoretical background knowledge based on research is needed. Theories on stability, however, are not completely based on mathematical solutions of the governing equations, but partly also on empirical tests. This raises requirements to education. Codes, on the other hand, are mainly meant for design, and the theory behind is hardly seen. The same applies to computer programs. As a consequence young engineers especially are not aware, what they really are calculating when dealing with stability problems.

The present paper briefly describes the theoretical and practical methods used in research, education and design and tries to compare them in such a way that the reader would be able to get an overall view on the entire field of stability problems. Examples are used to emphasise the practical side including codes.

## 2. THEORETICAL BACKGROUND TO COLUMN AND ARCH BUCKLING

### 2.1 Centrally loaded columns

The cornerstone of column theory is the Euler column, a straight, prismatic, pin-ended, perfectly centrally loaded strut that is slender enough to buckle without the stress at any point in the cross-section extending the proportional limit of the material.

The isolated column is a theoretical concept; it rarely exists in practice. Usually, a column forms part of a structural frame and its stability is interrelated with the stability of the entire structure. Now only the isolated column is of interest, because firstly, many structural design situations are idealized in such a way that elements can be considered as centrally loaded columns (*e.g.*, truss members, arches), and secondly, a centrally loaded column forms a limiting point in the mathematical space defining interaction between axial and flexural forces of a structural member. Thus an understanding of the strength of an individual centrally loaded columns is essential when developing a design criteria for compression members in general.

Column strength can be approximated either by using *critical load theory*, where the column is supposed to have mathematically perfect geometry, or by using the *theory of imperfect columns*, where a column and its load are slightly imperfect. The reason for this dual representation (*i.e.* perfect and imperfect) is that for practical purposes some types of columns (*e.g.* cold-formed steel and aluminium columns) can be idealized as perfect, while for other columns (*e.g.* hot-rolled or welded built-up structural steel columns) it is necessary to consider the effects of imperfections.

The research results are adequately reported by Timoshenko and Gere<sup>1</sup>, Beedle<sup>2</sup>, Galambos<sup>3</sup> and Ylinen<sup>4</sup>.

## 2.2 Arches

A member or structure curved in vertical plane and intended to resist external forces by mainly axial compression is called an arch. When the load acting on an arch increases, the arch loses its stability once a certain critical value of the load is attained. In the case of elastic structures under conservative loads, the critical load always corresponds either to a bifurcation point or to a stability limit point. Various possible load-displacement curves for arches, called as an equilibrium path, are schematically illustrated in Fig. 1 for in-plane instability and in Fig. 2 for out-of-plane instability, respectively. In both cases, the relative magnitude of the maximum loads in different cases (i), (ii), (iii) may vary in accordance with the respective loading conditions and structural properties.

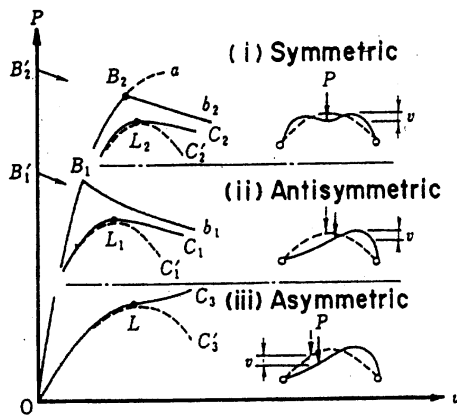


Figure 1: Load-deflection curves for in-plane instability

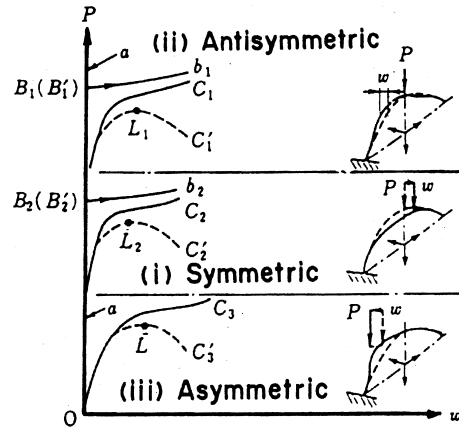


Figure 2: Load-deflection curves for out-of-plane instability

When the arch configuration and the loading conditions are symmetric with respect to the arch crown, the equilibrium path bifurcates from the direct deformation mode (curve  $a$ ) to the buckling deformation mode (curve  $b_1, b_2$ ) at the critical point  $B_1$  or  $B_2$ .

The buckling deformation is either symmetric (curve  $b_2$ ) or antisymmetric (curve  $b_1$ ) with respect to the crown. Which mode will occur under a smaller load depends on the type of the arch. For the in-plane problem of a 2-hinged or clamped arch an antisymmetric buckling occurs while for the out-of-plane problem in these cases a symmetric buckling is dominant. In the case of an asymmetric arch or a symmetric arch subjected to an asymmetric load, the load and deformation increase simultaneously until the maximum or limit load is reached. Beyond that point the system becomes unstable (curve  $c_3$ ).

Most of the earlier studies involve seeking of the critical load by approximating the maximum load  $L_1$  or  $L_2$  by means of an eigenvalue analysis. The eigenvalue analysis yields to the critical load  $B'_1$  or  $B'_2$  by disregarding the displacement before buckling, but when taking this displacement into account, it yields to the critical load  $B_1$  or  $B_2$  as a nonlinear problem.

The relevant research results are adequately reported by Timoshenko and Gere<sup>1</sup>, Beedle<sup>2</sup>, Galambos<sup>3</sup>, and Fukumoto<sup>5</sup>.



The values differ from each other because of the following reasons:

- The real axis of the arch differed from the theoretical one.
- In the calculations, the connectors were supposed to have constant rigidity, while in reality their buckling decreased their efficiency and resulted in increased moments and deformations.

Finally, the failure process under uniform load is shown in Fig. 5. The failure started due to rib buckling at the place where the shear force reached its maximum. The measured failure load intensity was  $570 \text{ N/m}^2$ . The corresponding calculated value was  $648 \text{ N/m}^2$ .

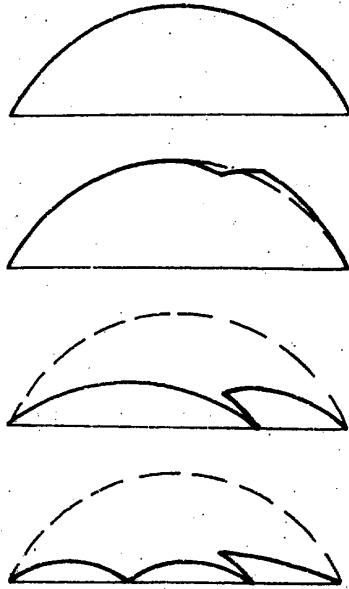


Figure 5: The process of collapse



Figure 6: The collapsed arch



Figure 7: The collapsed arch detail

#### 4. NUMERICAL ANALYSIS OF TWO SAMPLE BRIDGES

In this clause two numerically studied sample arch bridges are presented. The first example is a 78,95 m span Langer girder railway bridge with an orthotropic deck<sup>7</sup>. The second example is a 64,4 m span highway steel arch bridge with a steel-concrete composite stiffening girder<sup>8</sup>. The main dimensions of the bridges are shown in Fig. 8 and Fig. 9, respectively. It can be noted that in the previous case the arches are laterally connected to each other at three levels, but in the latter case they stand separately (individual arches). The bridges are subjected to loads of Structural Eurocodes. The vehicle models are LM71 and LM1 according to EN 1991-2<sup>9</sup>.

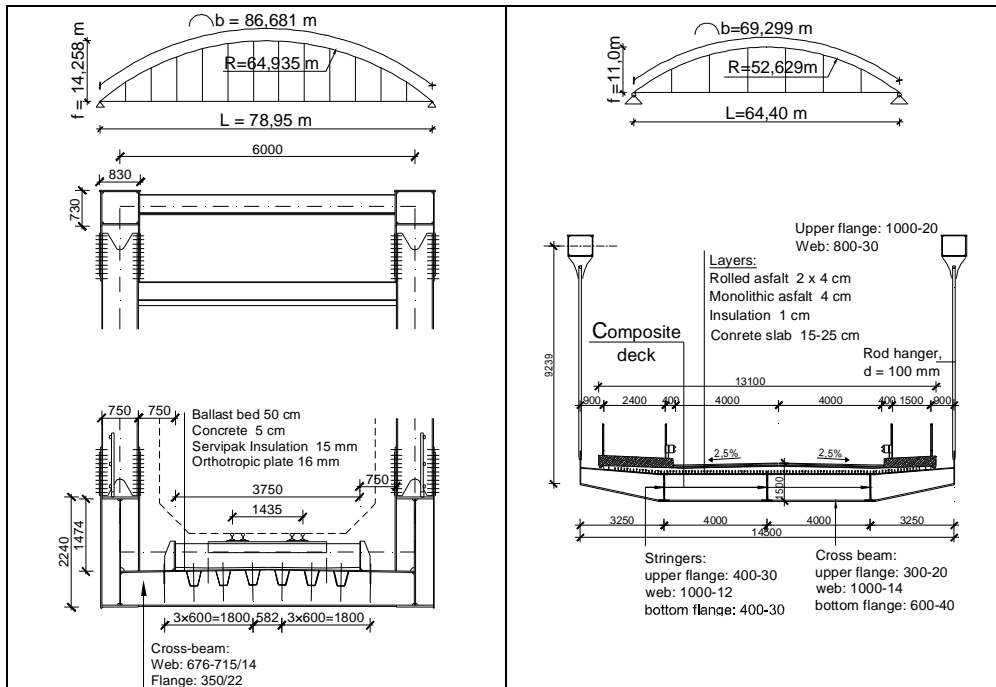


Figure 8: Langer girder railway bridge with an orthotropic deck

Figure 9: Highway steel arch bridge with steel-concrete composite stiffening girder

For calculation of the sample bridges two kinds of finite element models, Model 1 for the railway bridge and Model 2 for the highway bridge, are built up. The analysis is carried out using AXIS VM software program package developed for civil engineering purposes<sup>10</sup>. Each model is based on three-dimensional idealisation of the bridges. Linear material properties are used. For the steel-concrete composite deck in Model 2 the effective modulus method is applied. In Model 1 the orthotropic deck with the cross girders and the lower chord and in Model 2 the steel-concrete deck (concrete slab with connectors, stringers and cross beams) are modelled with shell elements, while in both models the arches (in case of Model 1 the upper lateral struts as well) and the hangers are built up by beam elements. The latter elements provide to examine the arches and auxiliaries during buckling analysis. As a result of finite element analyses, the average Euler critical normal forces  $N_{cr}$  of one arch element are 27,74 MN (Model 1) and 105,24 MN (Model 2), respectively.

## 5. BUCKLING ANALYSIS OF THE SAMPLE BRIDGES BY THE EUROCODE

The EN 1993-2 standard<sup>11</sup> provides two options to check the buckling of an arch or compressed members of bridges, namely method based on the determination of elastic critical parameter  $n_{cr}$ , or approximate design charts to determine the buckling length. In case of the first option the Euler critical normal force  $N_{cr}$  for the relevant compressed member using parameter  $n_{cr}$  can be determined ( $N_{cr} = n_{cr} \cdot N_{Ed}$ ).

So the relative slenderness parameter

$$\lambda = \sqrt{\frac{A \cdot f_y}{N_{cr}}}$$

can be obtained, where

$N_{Ed}$  is the normal force due to effects,

$A$  is the area of the compressed member and

$f_y$  is the yield strength of the material.

Finally, the reduction factor  $\chi$  as function of parameter  $\lambda$  and consequently the buckling force  $N_{b,Rd}$  can be determined and the condition  $N_{b,Rd} \geq N_{Ed}$  can be fulfilled. It is worth noting that in a structure for a relevant compressed member, the Euler critical normal force  $N_{cr}$  is always constant, *i.e.*  $N_{cr}$  does not depend on the magnitude of the load.

For the second option the Annex D of EN 1993-2 standard<sup>11</sup> presents design charts for the determination of buckling factors regarding to in-plane and out-of-plane buckling of the arch, which enables approximate calculations.

Using the average Euler critical normal forces  $N_{cr}$  along the arch or approximate design charts, the arch elements of the two bridges can be checked.

From the “exact” and the approximate calculations it can be concluded, that in the in-plane buckling analysis the results of the two methods mentioned above differ only slightly from each other. In the case of out-of-plane buckling, however, the differences are remarkable. This means that the approximate method, *i.e.* design charts, can be used only for preliminary calculations.

## 6. TEACHING OF STABILITY PROBLEMS

As it can be seen above, solving of stability problems has a great variety of possibilities. Even theoretically solid solutions do miss and that is why an overall picture of the different methods may remain unclear. This is even more obvious in teaching surroundings, where in a short time the theories as well as practical application methods should be introduced.

According to the long-term teaching experience of the present authors, simple pin-ended column's buckling theory based on Euler's solution should be the one to start with. This solution leads to an ordinary second order differential equation, whose solution with actual boundary conditions leads to a mathematically solid result. That is easily understood by young students, although the physical reality would remain somewhat unclear. After that the different boundary conditions of the same column should be introduced emphasising the difference between the theoretical and the real practical cases. Consequently the changes due to material yielding and the corresponding buckling curves used in codes should be introduced emphasising the semi-empirical result of the column buckling analysis.

Lateral buckling of straight beams was not discussed above, but because of its importance it should also be included in educational courses on the basic university level.

Finally, in-plane and out-of-plane buckling of arches are theoretically complicated subjects that commonly are left to be taught in advanced courses of structural mechanics. Here, more than elsewhere, sophisticated computer programs are involved even in simpler cases. Because such programs are not always available, stability control of arches often remains uncovered. In the case of big projects, however, such study is needed and carried out, as shown by the examples discussed in Chapter 4.

## 7. CODE WRITING

In many cases the codes are written in such a form that the real physical behaviour cannot be followed. An improvement would be, if the mathematical formulation of the code procedure and equations at least were presented in another way and the symbols familiar from theoretical studies were used. In lateral stability calculations at least, for instance, symbols known from the warping torsion theories should be used rather than physically unobvious calculation methods presented in codes internationally.

## 8. CONCLUSIONS

During the design of a real structure the buckling analysis can be carried out by using firstly the approximate methods presented in codes and secondly the so-called "exact" methods based on analytical or numerical analysis.

A stability process varies a lot depending on which method is used: analytical, numerical, experimental or design codes. Their differences make it difficult to get an overall view on them. They also lead to different results, which increases the engineers responsibility to verify the applicability of the solution. In all cases the model selection is of main importance.

For future code writing attention should be paid on theoretical studies of stability problems and international harmony.

## REFERENCES

- [1] Timoshenko, S.P. & Gere, J.M.: Theory of Elastic Stability, (second edition) McGraw-Hill book Company, New York, Toronto, London, 1961. 558 p.
- [2] Beedle, L.S. (editor-in-chief): Stability of Metal Structures; A World View, (second edition), Structural Stability Research Council, Bethlehem, USA, 1991. 980 p.
- [3] Galambos, T.V. (editor): Guide to Stability Design Criteria for Metal Structures, 5<sup>th</sup> edition, John Wiley and Sons, Inc., New York, 1998. 911 p.
- [4] Ylinen, A.: Kimmo- ja lujuusoppi (Strength of Materials). Part 2. 2<sup>nd</sup> editon. Porvoo 1970. 550 p. (In Finnish)
- [5] Fukomoto, Y. (editor): Structural Stability Design; Steel and Composite Structures Pergamon Press, 1997. 520 p.
- [6] Fernezelyi, S. & Ivanyi, M.: Tests Methodes for the Stability Analysis of Built-up Aluminium Arches, Proceedings, Regional Colloquium on Stability of Steel Structures, Hungary, Budapest-Balatonfüred, 19-21. October 1977. pp. 337-345.
- [7] Tokaji, S., Vasúti híd tervezése. (Design of a Railway Bridge). BME Diploma Work. Supervisor: Szabó, B. Budapest 2008. 150 p. (In Hungarian)
- [8] Siegler, B., Közúti híd tervezése. (Design of a Highway Bridge). BME Diploma Work. Supervisor: Szabó, B. Budapest 2009. 150 p. (In Hungarian)
- [9] EN 1991-2: Eurocode 1: Actions on structures. Part 2: Traffic loads on bridges.
- [10] INTERCAD LTD., AXIS VM user manual. Version 9. Budapest, Hungary .
- [11] EN 1993-2: Eurocode 3: Design of steel structures. . Part 2: Steel bridges.